Frequency Modulation of Resistance-Capacitance Oscillators*

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Summary—A method is described for direct-frequency modulation of a resistance-capacitance oscillator which is simpler and more stable than the beat oscillators formerly used. Spurious amplitude modulation is reduced to a negligible value without the use of limiters and without introducing appreciable harmonics in the output wave. Balanced control tubes prevent transients or signal frequencies from appearing in the output. Curves are given which provide an easy method of choosing the network and constants for any desired condition. The device is especially suited to facsimile transmission by subcarrier frequency modulation.

Introduction

THE USES of frequency-modulated subcarriers in facsimile and communications systems have greatly multiplied in the past few years.[1,2] In all of these instances the frequency swing is a sizable percentage of the carrier frequency, and cannot be obtained directly by reactance-tube control of an inductance-capacitance tank-circuit oscillator. The usual method has been to use reactance tube control of one or both tanks in a beat oscillator, thus obtaining a low-frequency carrier with the same frequency shift as the controlled oscillator.

The frequency-modulated resistance-capacitance oscillator[3] here described replaces the beat-oscillator system and has been found to be far more stable and considerably simpler in adjustment. The large frequency swings are obtained directly without heterodyning and the frequency drift usually associated with beat oscillators is thus avoided.

The frequency of oscillation of any resistance-capacitance oscillator is determined by the constants of the network, and therefore, changing any resistance or capacitance value will change the frequency. Replacing any of the resistive elements with a tube allows the frequency to be controlled by this tube.

There are two general classifications of resistance-capacitance oscillators, those having zero phase shift in the network,[4–6] and those having 180-degree phase shift in the network.[7] Tubes may be used to replace one or more of the resistors in oscillators of either type, though there are definite advantages in using oscillators with 180-degree phase-shift ladder networks. In either case, changing any single element of the network generally will change the network loss and give some undesired amplitude modulation in addition to the frequency modulation. These amplitude variations are more easily eliminated in the ladder network oscillator without resorting to automatic volume controls which limit the speed of response of the system. By proper choice of constants and operating conditions the amplitude modulation can be reduced to a negligible amount even with shifts as high as ± 40 per cent of the carrier frequency, and without introducing appreciable harmonic distortion.

The particular frequency-modulated oscillator used in one facsimile system[8] will be described first. Later, more general data on design will be given.

The Modulated Oscillator

The basic circuit of the oscillator, Fig. 1, is a four-step series-capacitor ladder network driven by a pentode T_1 and cathode follower T_2. If all network resistors were alike a voltage gain of 18.36 would be required to overcome the network loss and cause oscillation.[9] But in this case R_2 is made enough smaller than the other three resistors, R_1, R_3, and R_4, to make the network loss when R_2 is grounded about equal to the loss when R_2 is open-circuited. With R_2 open the fundamental frequency is 2000 cycles, and with R_2 grounded it is somewhat over 4000 cycles. Values of resistance in series with R_2

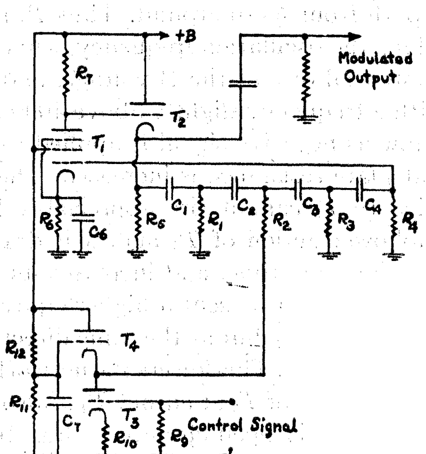

Fig. 1—Frequency-modulated resistance-capacitance oscillator.

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[1] R. E. Mathes and J. N. Whitaker, "Radio facsimile by subcarrier frequency modulation," *RCA Rev.*, vol. 4, pp. 131–154; October, 1939.

[2] Warren H. Bliss, "Use of subcarrier frequency modulation in communication systems," Proc. I.R.E., vol. 31, pp. 419–423; August, 1943.

[3] U. S. Patent 2,321,269, June 1943.

[4] C. K. Chang, "A frequency-modulated resistance-capacitance oscillator," Proc. I.R.E., vol. 31, pp. 22–25; January, 1943.

[5] H. H. Scott, "A new type of selective circuit and some applications," Proc. I.R.E., vol. 26, pp. 226–236; February, 1938.

[6] F. E. Terman, R. R. Buss, W. R. Hewlett, and F. C. Cahill, "Some applications of negative feedback with particular reference to laboratory equipment," Proc. I.R.E., vol. 27, pp. 649–655; October, 1939.

[7] E. L. Ginzton and L. M. Hollingsworth, "Phase-shift oscillators," Proc. I.R.E., vol. 29, pp. 43–49; February, 1941.

[8] U. S. Patent 2,326,740; August, 1943.

[9] This ratio is obtained from the equations of the network in Fig. 3A.

intermediate between zero and infinity will give frequencies between these limits. These varying values of resistance are furnished by the tube circuit T_3 and T_4.

The control circuit has the direct-current component of its output balanced out, so that changing the equivalent resistance in series with R_2 does not introduce into the network transients or modulating signal frequencies. The resistors R_{11} and R_{12} form a center tap on the B voltage, and T_4 is connected as the plate resistor of T_3. If T_3 is at zero bias, it draws full plate current

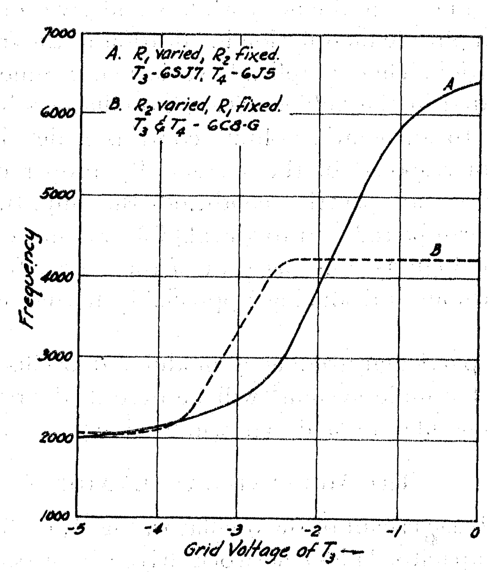

Fig. 2—Frequency-modulation characteristics of oscillator in Fig. 1.

through T_4, and the two tubes together form a low-resistance path from R_2 to ground. Thus R_2 is effectively grounded for the oscillation frequency, while at a direct-current potential of half the B voltage, and a four-step ladder with a frequency slightly above 4000 cycles is obtained. Now as negative signal is applied to T_3 grid, its equivalent plate resistance is increased. The plate voltage applied to T_3 cannot rise appreciably because the cathode-follower action of T_4 makes it draw less plate current at the same time, and increases its plate resistance. T_3 and T_4 then present a higher equivalent resistance in series with R_2, but at the same direct-current potential above ground. This lowers the network frequency. In the extreme case of T_3 at cutoff, T_4 is also cut off, and the return of R_2 is open-circuited, thus bringing the network to the 2000-cycle end of its range.

This control system can be used to vary any of the resistors in the network. The curves in Fig. 2 illustrate two conditions. For curve A the resistor R_1 was varied by a tube combination of a 6SJ7 for T_3 and a 6J5 for T_4. R_2, R_3, and R_4 were equal and all grounded. For curve B, resistor R_2 was varied and R_1 fixed. The control tubes in this second case were the two triodes of a 6C8-G tube.

The linearity of the modulation will be determined by which resistor or resistors are varied, and by the characteristic of T_3. Methods of making the linear range very wide are described later. In this case curve A is linear from 3000 to 5800 cycles, a swing of ± 32 per cent on a carrier of 4400 cycles. Curve B is linear from 2300 to 4000 cycles, or a swing of ± 27 per cent on a carrier of 3150 cycles.

In both cases the amplitude modulation is negligible over the linear range, and only a few per cent at the extreme ends of swing. This would not be true, however, except for the precautions described under "Amplitude Characteristics."

The speed of operation is apparently instantaneous. Oscilloscope observations have shown, for instance, that when using the system as for curve B, the oscillator can be shifted from 3000 to 4000 cycles, held on 4000 cycles for only 1 cycle, and then shifted back to 3000 cycles without any transients observed in the signal either before or after demodulation in a suitable discriminator. Such precise following of a square-wave signal makes the method very useful for facsimile transmission.

Frequency Ranges

A. Four-Step Ladder Network

The frequency range to be expected and the best operating conditions can be determined from calculated curves of the networks. Fig. 3A shows the four-step ladder as used in Fig. 1. All capacitors are assumed equal,

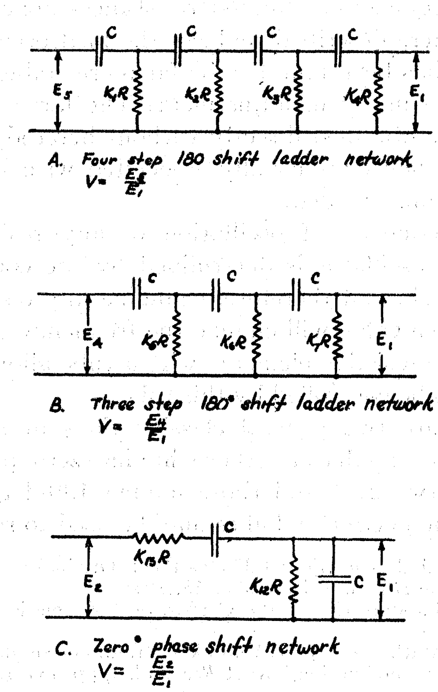

Fig. 3—Three types of resistance-capacitance networks.

and for simplicity in calculations R is assumed to be 1. Thus the complex equations for E_5/E_1 have only the variables X_c and K. Frequency is plotted as the numerical value of $1/X_c$, and to obtain actual cycles per second the F ordinate must be multiplied by $1/2\pi RC$. Under these conditions the equation for E_5/E_1 is

$$\frac{E_5}{E_1} = V = \frac{K_1K_2K_3K_4 - X^2(3K_1K_2 + 4K_1K_3 + 2K_1K_4 + 3K_2K_3 + 2K_2K_4 + K_3K_4) + X^4}{K_1K_2K_3K_4} + j\left[\frac{X^3(2K_1 + 2K_2 + 2K_3 + K_4) - X(4K_1K_2K_3 + 3K_1K_2K_4 + 2K_1K_3K_4 + K_2K_3K_4)}{K_1K_2K_3K_4}\right]. \quad (1)$$

If the first resistor only is varied, then $K_2 = K_3 = K_4 = 1$,

$$\text{therefore } V_1 = \left[\frac{K_1 - X^2(9K_1 + 6) + X^4}{K_1}\right] + j\left[\frac{X^3(2K_1 + 5) - X(9K_1 + 1)}{K_1}\right]. \quad (2)$$

When oscillating, the network shift is 180 degrees, and the (j) term, therefore, must be zero.

$$\text{Therefore } X^2 = (9K_1 + 1)/(2K_1 + 5) \quad (3)$$

$$\text{and } F_1 = 1/X = \sqrt{(2K_1 + 5)/(9K_1 + 1)}. \quad (4)$$

This is plotted as F_1 in Fig. 4, and shows the relative frequency for which the network phase shift is 180 degrees as K is varied.

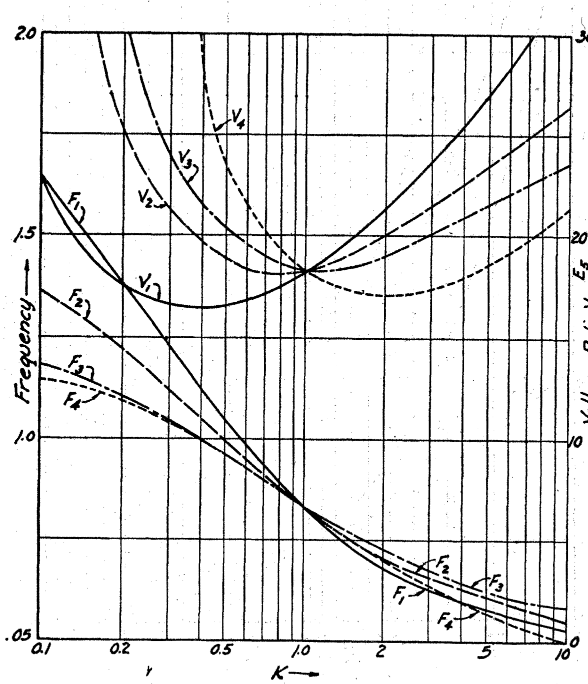

Fig. 4—Frequency and voltage ratio characteristics of four-step ladder network in Fig. 3A.

By placing the value of X^2 from (3) in (2), a plot is obtained of the voltage ratio of input to output (V_1) as K varies. This is also plotted in Fig. 4 as V_1. Actually it is a negative number, but plotted positive here for convenience.

In a similar manner, if R_2 only is varied, $K_1 = K_3 = K_4 = 1$, and a new equation for V_2 is obtained. From this the curves F_2 and V_2 in Fig. 4 are plotted F_3, V_3 and F_4, V_4 are similarly derived from (1) by using either K_3 or K_4 as the variable.

B. Three-Step Ladder Network

The ladder network may be used with only three steps if desired, as in Fig. 3B. With this network, and similar conditions of $R = 1$ and all capacitances equal, the equation for $V = E_4/E_1$ becomes

$$V = \left[\frac{K_5K_6K_7 - X^2(2K_5 + 2K_6 + K_7)}{K_5K_6K_7}\right] + j\left[\frac{X^3 - X(3K_5K_6 + 2K_5K_7 + K_6K_7)}{K_5K_6K_7}\right]. \quad (5)$$

As before, to vary the first resistor only make $K_6 = K_7 = 1$, then

$$V_5 = \left[\frac{K_5 - X^2(3 + 2K_5)}{K_5}\right] + j\left[\frac{X^3 - X(5K_5 + 1}{K_5}\right]. \quad (6)$$

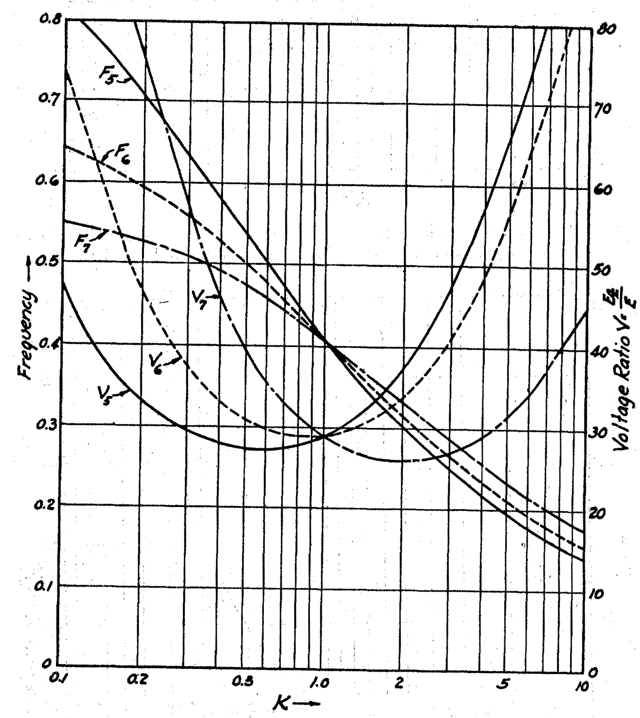

Fig. 5—Frequency and voltage ratio characteristics of three-step ladder network in Fig. 3B.

When oscillating, the shift is 180 degrees and the (j) term zero.

$$X^2 = 5K_5 + 1 \quad (7)$$

$$\text{and } F_5 = 1/X = 1/\sqrt{5K_5 + 1}. \quad (8)$$

This is plotted as F_5 in Fig. 5. By substituting this value into (6) the plot of V_5 is obtained. The curves V_6, F_6 and V_7, F_7 are obtained by using either K_6 or K_7 as the variable in (5).

The curves in Figs. 4 and 5 will give the frequency range, and the required change in amplifier gain to cover this range, when varying any one resistor of either a four-step or a three-step ladder. In all cases the network-voltage ratio passes through a broad minimum value, an operating point where wide frequency shifts may be obtained with only a small amount of amplitude modulation. These points of minimum loss generally occur at the value of K for which the rate of change of F with K is largest, a fortunate condition for obtaining large frequency changes.

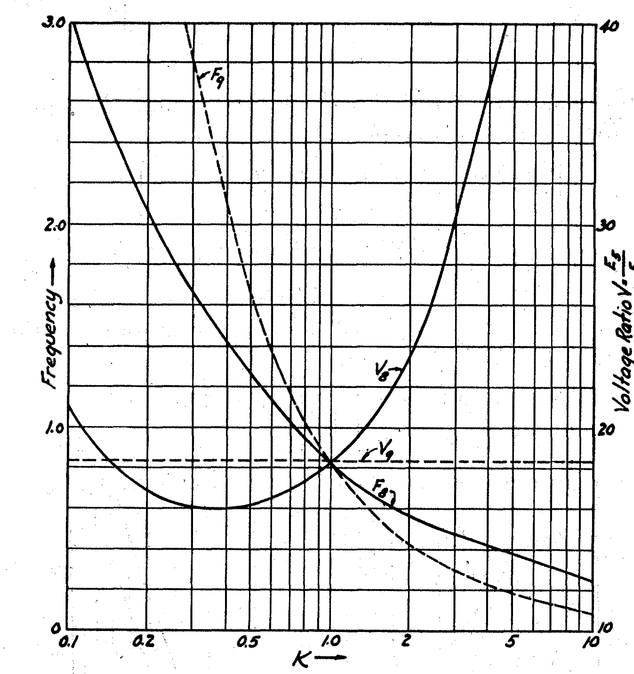

Fig. 6—Effect of varying first and second resistors together, and of all four resistors together, of network in Fig. 3A.

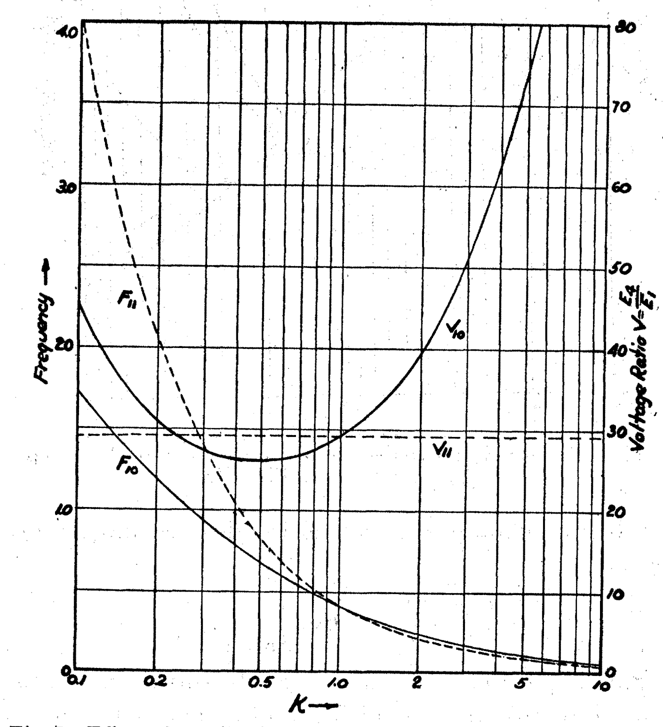

Fig. 7—Effect of varying first and second resistors together, and of all three resistors together, of network in Fig. 3B.

It is possible to control more than one resistor and obtain still wider frequency swings without undue amplitude change. If the first two resistors in the four-step ladder are changed, the curves F_8, V_8 of Fig. 6 are obtained. When all resistors are varied together the voltage ratio remains constant at 18.36, and the possible swing is theoretically infinite. This is shown by curves F_9, V_9 in Fig. 6.

When the first two resistors of the three-step ladder are controlled, the curves F_{10}, V_{10} of Fig. 7 are obtained; and when all three are controlled, F_{11}, V_{11}. In this latter case the loss ratio remains constant at 29.

C. The Zero Shift Network

For the sake of comparison, similar curves for the zero phase-shift network in Fig. 3C are derived and plotted[10] in Fig. 8. Here again $R=1$, and the frequency ordinate must be multiplied by $1/2\pi RC$ to obtain cycles per second. The fundamental equation for $V=E_2/E_1$ is, in this case,

$$V = \frac{X(K_{13} + 2K_{12}) + jK_{12}K_{13} - jX^2}{K_{12}X}. \tag{9}$$

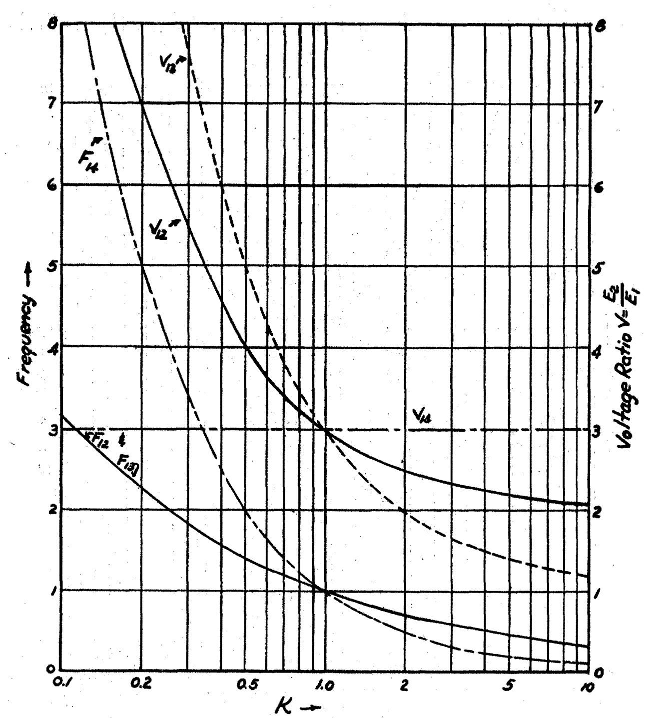

Fig. 8—Frequency and voltage ratio characteristics of network in Fig. 3C when either or both resistors are varied.

For oscillation, the shift is 0 degrees, so the (j) terms must cancel:

$$K_{12}K_{13} = X^2. \tag{10}$$

If K_{12} alone is varied and $K_{13}=1$:

$$X^2 = K_{12}$$

$$F_{12} = 1/\sqrt{K_{12}} \tag{11}$$

and the voltage ratio becomes

$$V_{12} = 1 + 2K_{12}/K_{12}. \tag{12}$$

Similarly, V_{13} and F_{13} are obtained as

$$F_{13} = 1/\sqrt{K_{13}} \quad \text{(same as } F_{12}\text{)} \tag{13}$$

and

$$V_{13} = K_{13} + 2/K_{13} \tag{14}$$

and if both resistors are varied together, the equations

[10] See footnote references 4, 5, and 6 for circuits of this type of oscillator.

for F_{14} and V_{14} are obtained:

$$F_{14} = 1/K_{14} \tag{15}$$

$$V_{14} = 3. \tag{16}$$

It can be seen that neither of the amplitude curves V_{12} and V_{13} has a minimum value, but approach, respectively, the limits of 2 and 1. Thus, there is no operating point where a change in frequency may be obtained without at the same time introducing some amplitude change.

Experience has shown that the circuit in Fig. 1 can be operated without noticeable amplitude modulation over ranges where the network loss does not change over 20 per cent. (See topic on Amplitude Characteristics.) The results of the curves in Figs. 4, 5, 6, 7, and 8, therefore, can be tabulated to compare the frequency shift at an amplitude change of 20 per cent. This tabulation also indicates the center operating position, in relative frequency, and the central value of K, for determining the ratio of control-tube-circuit resistance to the network resistors.

TABLE I

FREQUENCY SHIFT WHEN 20 PER CENT CHANGE IN NETWORK VOLTAGE RATIO IS ALLOWED

Resistor Varied	Voltage Ratio		Mid-Frequency	Percentage Frequency Shift on Mid-Frequency	Mid-Frequency Value of K
	Minimum	Maximum			
A. Four-Step Ladder					
1st Res. (K_1)	16.6	19.92	1.14	±38.5	0.38
2nd Res. (K_2)	18.2	21.84	0.87	±31.0	0.63
3rd Res. (K_3)	18.33	22.0	0.82	±24.4	0.66
4th Res. (K_4)	17.0	20.4	0.67	±22.4	2.60
1st & 2nd (K_8)	16.0	19.2	1.70	±56.5	0.29
B. Three-Step Ladder					
1st Res. (K_5)	27.0	32.4	0.53	±32.0	0.50
2nd Res. (K_6)	29.0	34.8	0.43	±27.0	0.75
3rd Res. (K_7)	26.0	31.2	0.333	±30.0	2.0
1st & 2nd (K_{10})	26.0	31.2	0.75	±55.3	0.40
C. Zero Phase-Shift Network					
*1st Res. (K_{13})	2.7	3.3	1.0	± 8.0	1.0
*1st Res. (K_{13})	1.2	1.47	0.398	±26.0	6.5
*2nd Res. (K_{12})	2.7	3.3	1.0	±15.0	1.0
*2nd Res. (K_{12})	2.1	2.56	0.566	±44.2	3.0

* For these curves no minimum value of V exists, so two values are given, one for the center of the range where K is 1, and the other where K is 10 at the low frequency end of the swing.

This tabulation will indicate the amount of frequency swing that can be obtained with a 20 per cent change in amplitude, but it does not indicate how linear these swings will be.

Amplitude Characteristics and Harmonic Distortion

Narrow frequency shifts can be obtained with very little amplitude change if the center operating positions are chosen to correspond to the minimum V values. When wide limits of shift are desired, some precautions are necessary. There are two obvious methods of reducing this amplitude change, by limiters or by some form of automatic volume control. The second method is applicable where the modulating frequencies are a very small percentage of the carrier frequency. When the modulating frequencies approach the frequency of the carrier, then the automatic control must become a cycle-by-cycle device, or in reality a limiter, if the higher modulating frequencies are not suppressed. In the case of facsimile, where the modulating frequencies are as high as 30 or 40 per cent of the mid-carrier frequency, limiting is the logical method.

One of the attributes of the ladder-type oscillators, as compared to the zero shift type, is its ability to limit without causing high harmonic distortion, or squaring of the resulting output. This limiting is accomplished in the oscillator-tube circuit itself. If the curve F_1 is used, and it is desired to swing the full range of ±38.5 per cent, or from 1.54 to 0.74, the tube-driver circuit is adjusted to give the maximum gain of 19.92. The circuit then will oscillate at the frequencies of 1.54 or 0.74 at full amplitude. When the center frequency of 1.14 is reached the gain will be 20 per cent too high, and some squaring of the wave should result. However, the network will have 180 degrees shift only at the fundamental frequency and will thus serve as its own filter. Higher harmonics will pass through with less phase shift and tend to cancel by degeneration. Lower frequencies than the fundamental will be suppressed by attenuation. Thus, as the network is varied to shift frequency its filtering action is also changed to be most effective at the generated frequency. External filters to remove distortion become unnecessary.

This is not true with the zero shift type of a resistance-capacitance oscillator. Here the output squares up very rapidly with a small amount of overdrive, and the frequency falls. This is due to the low order of filtering in the network. In this case, reasonably constant amplitude is necessary to preserve frequency stability and prevent distortion. The high shifts of ±26 and ±44.2 per cent as tabulated for this network probably cannot be realized except by using automatic volume control, or by allowing the oscillator to overdrive at the higher frequencies, and then filtering the harmonics from the output.[11] When overdriven too far, the circuit begins to function as a multivibrator and frequency stability is lost.

Linearity

In the tube circuit furnishing the variable resistance, considering the tube T_4 as a perfect cathode follower, the resistance inserted will be half that of either tube. The usual method of expressing the tube characteristics in a formula is

$$i_p = (E_p + \mu E_g)/R_{p0}. \tag{17}$$

The variable resistance inserted, therefore, will be

$$R_v = E_p R_{p0}/2(E_p + \mu Eg). \tag{18}$$

In this case, due to the action of T_4, E_p is always the same and equal to one half E_B.

Therefore, $$R_v = E_B R_{p0}/2(E_B + 2\mu Eg). \tag{19}$$

When all of the resistors of a network are varied at the same time, the frequency is proportional to $1/K$.

[11] It should be noted that in some facsimile uses the range is over two to one from maximum to minimum frequency. Thus, any external filtering of the second harmonic of the lowest frequency would also eliminate the upper-frequency fundamental, and introduce serious amplitude distortion.

Therefore, if R_v is made equal to K by the proportionality constant T

$$F_4 = T \cdot 2(E_B + 2\mu Eg)/E_B R_{p_0}. \tag{20}$$

Thus, frequency is a linear function with the control voltage on the grid of T_3.

When one or two resistors only are varied, the relationship does not hold, but the $K \cdot F$ curve of the network can usually be made to follow closely the $R_p \cdot Eg$ curve of the tube over wide ranges. Referring to the experimental curves in Fig. 2, it can be seen that the linear portion of B occupies about 80 per cent of the total swing. In certain experimental modulated oscillators this percentage was increased to over 90 per cent by using a pentode for T_3.

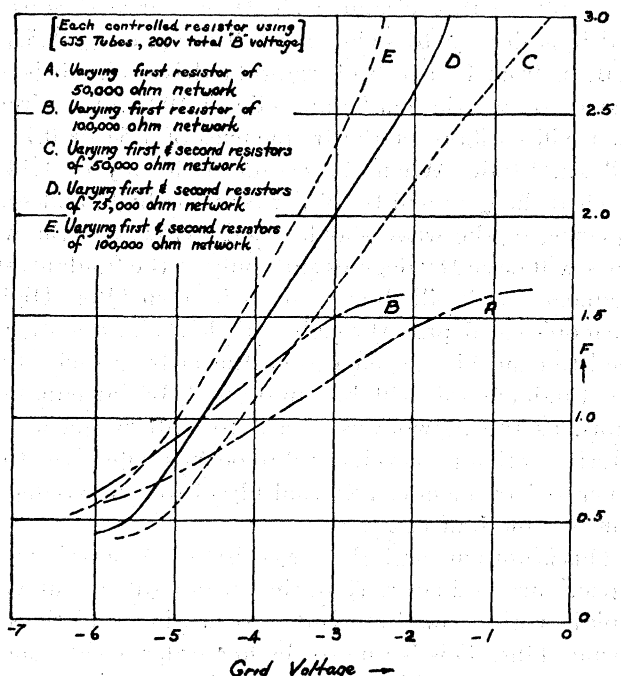

Fig. 9—Calculated characteristics using 6J5 control tubes to vary one or two resistors of network in Fig. 3A.

Fig. 9 illustrates how this fitting of tube curve and network may be carried out to obtain a frequency swing linear with grid voltage. For curve A, the network resistance in each step is assumed to be 50,000 ohms, and the two control tubes T_3 and T_4 are 6J5's. The total B voltage is assumed as 200 volts. If only the first resistor is variable, then the curve F_1, Fig. 4, is the network characteristic, and the points on curve A are determined as follows: At the point where F is 1, from curve F_1, Fig. 4, the value of K for the variable resistor is found to be 0.57. The network resistors that are not varied were assumed 50,000 ohms, so the two tubes (together) of the control must show K (50,000) or 28,500 ohms. As the two tubes are effectively in parallel, each tube must have twice this or 57,000 ohms actual resistance. With a total B supply of 200 volts, each tube then draws $200/2 \times 57{,}000$ or 1.755 milliamperes. The grid voltage necessary for this plate current with 100 volts on on the plate is read from tube charts or actual test values, -3.8 volts in this case. This process is repeated for as many points as desired. The curves where two resistors are varied are obtained in the same manner but using the F_8 curve in Fig. 6 to represent the network. Both A and B curves are S shaped, but B has a considerably more nearly straight center section. This indicates the 6J5-tube curve is better suited in the low-current region.

The curves C, D, and E show how the correct network resistance can be chosen for greatest linearity when two resistors are varied. Curve C has a long S shape, but is much more nearly straight than was found for only one variable resistor as in A. D is almost perfect over the range of from $F=0.5$ to $F=2.5$, a shift of ± 67 per cent about a center frequency of 1.5. Further increase in the network resistors gives line E, which has shifted to a hyperbolic shape and is curved throughout its length.

In using the curve D, these limits of linearity, $F=0.5$ to $F=2.5$, are put back into the voltage ratio curve V_8, Fig. 6, to see if this swing can be obtained without amplitude modulation. It is found that the $F=2.5$ end can be reached with a gain increase of only 14.4 per cent over the minimum value, but the $F=0.5$ end of the sweep requires a gain increase of 87 per cent. Obviously, this lowest frequency cannot be reached without more amplitude modulation than can be corrected by the natural limiting. The low-frequency end of the swing should be restricted to about $F=0.8$ for a symmetrical mid-carrier position. In this manner a linear swing of ± 51.5 per cent can be realized with only 14.4 per cent increase in gain, a value easily within the range of the limiting and, therefore, not causing distortion.

Conclusions

Direct-frequency modulation of resistance-capacitance oscillators is shown to be a very practical means of obtaining large swings on relatively low frequency carriers. The method is especially adaptable for facsimile and similar communication systems.

The circuits are simpler than the beat oscillators formerly used, and are much more stable in adjustment. Spurious amplitude modulation can be reduced to a negligible amount by choice of the proper network and constants, and the harmonic content of the output signal is low. Modulation of the resistance-capacitance oscillator is accomplished by a pair of control tubes so balanced that no transients or components of the original signal appear in the output.

Facsimile systems have been put in operation with these circuits with frequency swings of 500 to 1500 cycles, 2000 to 4000 cycles, 4000 to 9000 cycles, and 16,000 to 24,000 cycles.

Design of such systems has been simplified by providing the F versus K curves. These facilitate the choice of network constants and control tubes for any desired frequency swing.